

# Synchronized Collective Behavior via Low-cost Communication

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An important natural phenomenon surfaces that satisfactory synchronization of self-driven particles can be achieved via sharply reduced communication cost, especially for high density particle groups with low external noise. Statistical numerical evidence illustrates that a highly efficient manner is to distribute the communication messages as evenly as possible along the whole dynamic process, since it minimizes the communication redundancy. More surprisingly, it is discovered that there exist some abnormal regions where moderately decreasing the communication cost can even improve the synchronization performance. A phase diagram on the noise-density parameter space is given, where the dynamical behaviors can be divided into three qualitatively different phases: *normal phase* where better synchronization corresponds to higher communication cost, *abnormal phase* where moderately decreasing communication cost could even improve the synchronization, and the *disordered phase* where no coherence among individuals is observed.

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Over the last decade or so, physicists have been looking for common, possibly universal, features of the collective behaviors of animals, bacteria, cells, molecular motors, as well as driven granular objects. Thus, the collective motion of a group of autonomous particles is a subject of intensive research that has potential applications in biology, physics and engineering. One of the most remarkable characteristics of systems such as a flock of birds, a school of fish, or a swarm of locusts is the emergence of states of collective order in which the particles move in the same direction, i.e. *ordered state* [1, 2, 3], despite the fact that the interactions are (presumably) of short range. Moreover, this ordered state seeking problem can be further generalized to a consensus problem [4], i.e. groups of self-propelled particles agreeing upon certain quantities of interest like attitude, position, temperature, voltage and so on. Distributed computation based on solving consensus problems has direct implications on sensor network data fusion, load balancing, swarms/flocks, unmanned air vehicles (UAVs), attitude alignment satellite clusters, congestion control of communication networks, multi-agent formation control and so on [5, 6, 7].

In Ref. [1], a dynamical model describing the collective motion is proposed in a system of self-propelled particles. Due to its simplicity yet efficiency, this so-called Vicsek model has been drawing more and more attention recently and gaining increased popularity from both physics and engineering communities [2, 3, 8, 9, 10, 11, 12, 13, 14, 15]. In the Vicsek model each particle tends to move in the average direction of motions of its neighbors while being simultaneously subjected to noise. As the amplitude of the noise increases the system undergoes a phase transition from an *ordered state* in which the particles move in the same direction, to a *disordered state* in which the particles move independently in random directions. Grégoire and Chaté [2] modified the Vicsek model by changing the way in which the noise

is introduced into the group. By this means, the phase transition is switched from second to first order. More recently, in order to stabilize flocks/swarms, Gazi-Passino [10] and Moreau [11] developed two alternative models, i.e. the Attraction/Repulsion (A/R) model and the linearized model, respectively. The former yields a cohesive swarm with bounded size in a finite time, while the latter can guarantee the convergence of all the particles' states to a common one with *complete communication*, i.e. sending messages all along.

In brief, based on complete communication, most of the previous models of self-propelled particle groups yield many attractive characteristics like convergence, ordered state, consensus, rendezvous, cohesion, robustness, etc. However, in this Letter, an important phenomenon is discovered that complete communication is not the most efficient manner. For many kinds of self-propelled particle groups and natural swarms/flocks/schools, satisfactory ordered state performances can still be achieved with sharply reduced communication cost. Secondly, even more surprisingly, there exist some abnormal regions in the density-noise space where moderately reducing the communication cost can help increase the performance. A general physical picture behind our finding is as follows: in abundant natural bio-groups composed of animals, bacteria, cells and so on, each particle does not send messages throughout the whole process, but now and then in some suitable manner, which is called *partial communication*. Some close examples can be found in firefly groups, deep-sea luminous fish schools and so on. Each particle uses light signal with limited power to guide the others, and just flashes at some suitable discrete times to save energy, which yields satisfactory collective performances. Other than the above mentioned natural phenomena, our work is also partially inspired by Ref. [12], in which it is revealed that the larger the group the smaller the proportion of informed individuals needed to guide the whole group, and that only a very small propor-

tion of informed individuals is required to achieve great accuracy. In addition, we found the role of information redundancy on the present model: The higher the redundancy, the worse the synchronization performance. Therefore, a highly efficient manner is to distribute the communication messages as evenly as possible along the whole dynamic process. From an industrial application point of view, the phenomena and strategies reported in this Letter may be applicable in some relevant prevailing engineering areas like autonomous robot formations, sensor networks, UAVs and so on. Since each particle in these groups has just limited power to send messages, partial communication is required to save energy [5, 6, 7].

Due to its popularity, we will focus our simulation and investigation on the Vicsek model [1]. In this model, the velocities  $\{\mathbf{v}_i\}$  of  $N$  particles are determined simultaneously at each time step, and the position of the  $i$ th particle is updated according to  $\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t$ . Here the velocity of a particle  $\mathbf{v}_i(t+1)$  is constructed to have an absolute value  $v$  and a direction given by the angle  $\theta(t+1)$ . This angle is obtained from the expression

$$\theta(t+1) = \langle \theta(t) \rangle_r + \Delta\theta \quad (1)$$

where  $\langle \theta(t) \rangle_r$  denotes the average direction of the velocities of particles (including particle  $i$ ) being within a circle of radius  $r$  surrounding the given particle  $i$ . The average direction is given by

$$\langle \theta(t) \rangle_r = \arctan [\langle \sin(\theta(t)) \rangle_r / \langle \cos(\theta(t)) \rangle_r] \quad (2)$$

where  $\langle \sin(\theta(t)) \rangle_r$  and  $\langle \cos(\theta(t)) \rangle_r$  denote the average sine and cosine values of the velocities respectively, and  $\Delta\theta$  represents a random noise obeying a uniform distribution in the interval  $[-\eta/2, \eta/2]$ . In accordance with the Vicsek model, we use the same settings as in Ref. [1], i.e.  $r = 1$ ,  $v = 0.03$  and  $N = 300$ , and employ the absolute value of the average normalized velocity  $v_a = |\sum_{i=1}^N \mathbf{v}_i| / (Nv)$  as the performance index. The velocity  $v_a$  is approximately zero if the direction of motion of the individual particles is distributed randomly, while for the coherently moving phase (with ordered direction of velocities)  $v_a \simeq 1$ . Note that the linear size  $L$  of a square shaped cell determines the density  $\rho = N/L^2$ , and in all the simulations we use 1000 runs and  $M = 500$  running steps for each run.

For partial communication, only some of the particles will broadcast its position and velocity at each time step. The communication cost  $p$  is measured by the average number of broadcasting particles over the total number of particles at each time step. To investigate partial communication and find a highly efficient manner, we compare three communication manners, namely, *random*, *continuous* and *supervised communication*. The random communication manner (resp. the continuous communication manner) demands that each particle send  $p \cdot M$  messages reporting its position and velocity randomly (resp. continuously with randomly selected beginning step). The supervised manner is an intelligent one, in which each

particle calculates the average direction of its broadcasting neighborhood at each step. When the angular difference between its and neighboring directions surpasses an angular threshold  $\theta_t$ , it will broadcast its position and velocity to its neighbors. Obviously, the communication cost increases with decreasing  $\theta_t$ .

First, these three protocols are compared for a high density particle group ( $L = 5$ ) without noise (i.e.  $\Delta\theta = 0$  in Eq.(1)) in Fig. 1(a). The first attractive characteristic of the random manner is that satisfactory  $v_a$  can be yielded with sharply reduced  $p$  (e.g. no less than 95% of  $v_a$  of the complete communication for  $p \approx 2\%$ ). To further investigate the influence of the density  $\rho$  on the performance  $v_a$ , we have done simulations for a medium density case ( $L = 15$ ) and a low density case ( $L = 25$ ) (see Fig. 1(b) and Fig. 1(c), respectively). Comparing the performances for the random communication alone across Fig. 1(a)–(c), one can observe that, when the density  $\rho$  decreases, to achieve the same satisfactory  $v_a$  a higher communication cost  $p$  is required. Similar conclusions can also be drawn for the continuous and supervised protocols. Next, we investigate the effect of external noise by adding low and high noises in Eq. (1). As shown in Fig. 1(d)–(f), the noise has a more intensive effect on the performance than density since the influence of the noise is more direct. It can be seen that the random manner is the best among the three communication manners.

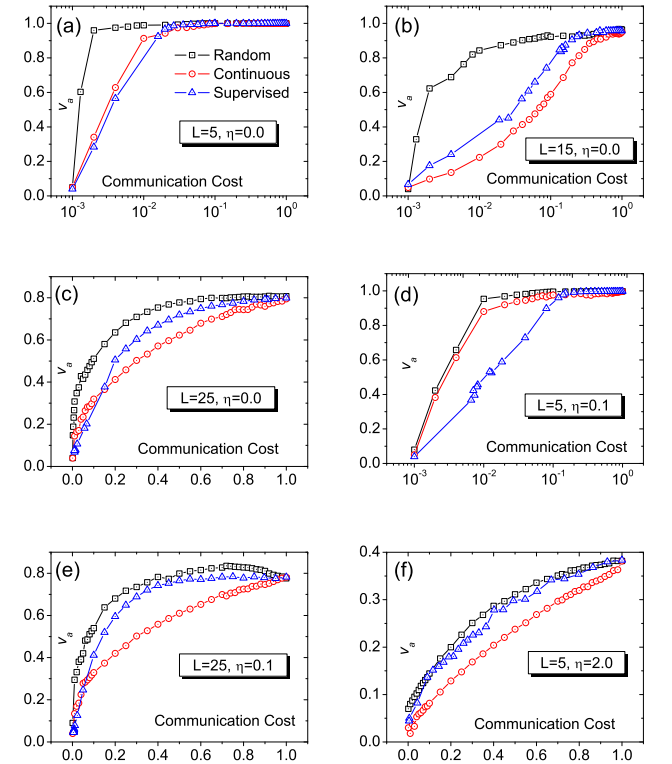


FIG. 1: (color online) Synchronized performance index  $v_a$  with respect to communication cost  $p$  for random (squares), continuous (circles), and supervised (triangles) manners.

The physical reason of achieving synchronized performances via very low communication cost may be: due to the high density (e.g.  $L = 5$ ), on average there are enough particles inside the radius  $r$  of each particle, and the combination of the sparse messages of each one of the plentiful neighbors constitutes an abundant information flow which can guide each particle to the right direction. If the density of particles is decreased (e.g.  $L = 15, 25$ ),  $p$  should be increased to compensate for the deficiency of the neighboring guidance information. As to why the random protocol is the best one among the three, the explanation may be that communication manners distributing messages more evenly are more efficient due to their reduced redundancy. In detail, in the continuous manner, when one message sent by a particle can guide its neighbors along the right direction, its succeeding messages do little to help the group performance. In this sense, these subsequent messages become redundant, and the efficiency is thus decreased substantially. On the other hand, since it is very natural to have a lurking suspicion that a more intelligent communication protocol implies better performance, it is surprising that the random manner is superior to the supervised manner. This fact should also be accredited to the communication redundancy. Statistical simulation shows that when a particle sends a message to avoid deviation, its neighbors are apt to send messages simultaneously. Thus it is of high probability that almost all the particles send messages at the same time. This supposition is supported by the simulations which show that the messages mostly aggregate at the very beginning of the whole procedure. As a result, communication redundancy is inevitable. In brief, it is reasonable to deduce that the best manner is to distribute the communication messages as evenly as possible along the whole dynamic process, since it minimizes the communication redundancy.

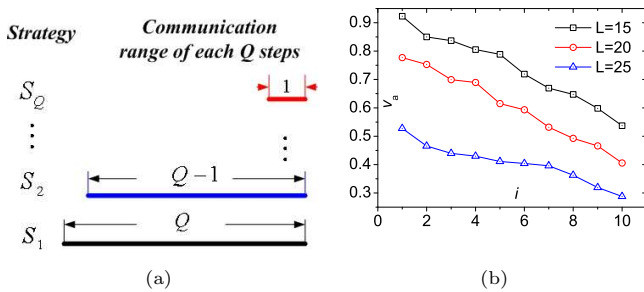


FIG. 2: (color online) (a) Illustration of different communication strategies, and (b) performance of strategy  $S_i$  ( $i = 1, 2, \dots, Q$ ). Here we set  $Q = 10$ .

We propose a toy model to demonstrate that more evenly distributed communication leads to better performance. As shown in Fig. 2(a), with a give integer  $Q$ , for strategy  $S_i$  ( $1 \leq i \leq Q$ ), each particle sends its messages with probability  $1/(Q - i + 1)$  if the current time step  $t$  satisfies the inequality  $t \bmod Q \geq i - 1$ . It is obvious that the communication cost  $p$  equals  $1/Q$  for each strategy

$S_i$  ( $i = 1, 2, \dots, Q$ ) and the communication redundancy increases with increasing  $i$ . From the performance comparison in Fig. 2(b), we can observe that  $v_a$  decreases with increasing communication redundancy.

Furthermore, a surprising phenomenon is observed that, in the case of the random manner with noise (e.g.  $L = 25, \eta = 0.1$  in Fig. 1(e)), there exist an abnormal region where moderately reducing  $p$  might even increase  $v_a$ . Thus, for the groups working in this region, each particle can use much less communication power to gain even better performance. The applaudable physical rule behind this astonishing phenomenon may be: in some suitable areas of the density-noise space, the influence of noise defeats the counterpart of the neighboring communications but it has not yet reached the extent of totally disordering the system dynamics, thus in some range of  $p$  (e.g.  $0.5 \leq p \leq 1$ ) more communication means propagating more errors. Consequently, partial communication outperforms complete communication. This rule can be very useful in plentiful industrial applications, since more benefits can be achieved with less communication cost in some working conditions. However, note that this phenomenon is only found in the random communication. As to the continuous and supervised manners, their communication redundancy is too much to arouse this abnormal phenomenon.

To illustrate the abnormal phenomenon of the random manner more vividly, first we provide several typical abnormal cases ( $L = 25, \eta = 0.1$ ;  $L = 6, \eta = 2$ ;  $L = 10, \eta = 2$  and  $L = 9, \eta = 1$ ) and mark their corresponding abnormal values by red points in Fig. 3. We sketch the diagram in Fig. 4 where the density-noise space is divided into three regions, namely abnormal, normal and disordered regions denoted by red, blue and green colors, respectively. Here, the disordered region represents the density-noise combinations with which the performance  $v_a$  remains at a very low random value no matter what  $p$  is. Furthermore, the intensity of the color represents the likelihood of the occurrence of each phenomenon. For instance, the very inner part of the abnormal region is marked by darker red color than the boundary, which means that in this central part the abnormal phenomenon is more likely to occur. The reasonableness of these regions can be validated by some simple arguments as follows. There is no abnormal phenomenon in noise-free cases, thus the line of  $\eta = 0$  always belongs to normal region; for  $\rho = \eta = 0$ , there is no particle, therefore the origin point is the only intersection of the three regions. More importantly, in the abnormal region (see red part of Fig. 4), the intensity of the noise has been increased to defeat the influence of the neighboring communication. However, if the noise is enhanced too quickly, then the system will enter the disordered state (see the green part of Fig. 4). Using such density-noise space diagrams, one can tell whether the current working condition of the network is in the abnormal region or not. If so, one can estimate how much communication energy can be saved to yield better performance than complete

communication. In this sense, the discovery of such abnormal regions will be valuable in abundant industrial applications.

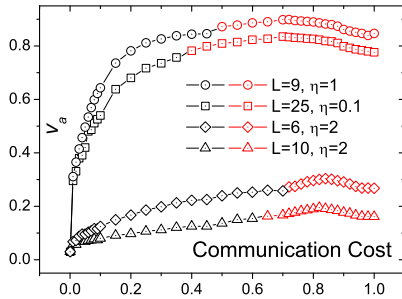


FIG. 3: (color online) Abnormal cases of random manner, red points denote abnormal phenomena.

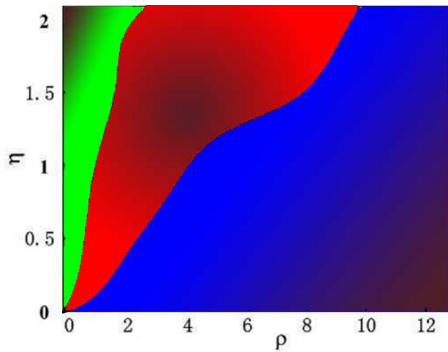


FIG. 4: (color online) Phase diagram: Abnormal (red), normal (blue) and disordered (green) regions.

In summary, we have numerically analyzed the collec-

tive dynamics of self-propelled particle groups via partial communication and found that: (i) Ordered state performance can be achieved with fairly low communication cost. When the density is high, for noise-free or low-noise cases, just a very small proportion of communication can produce satisfactory performances; in other words, almost no benefit can be gained by increasing the communication cost when  $p$  exceeds a very small value. (ii) There exist an abnormal region in the density-noise space, in which moderately reducing the communication cost can even improve the performance. (iii) More evenly distributed communication is superior.

To verify the universality of these conclusions, we have also applied the rule of partial communication to another two popular models of self-propelled particles, the A/R model [10] and the Moreau model [11]. The corresponding results also strongly suggest that complete communication is not always optimal when taking into consideration both the performance index and communication cost. For natural science, the contribution of this work is to explain why the particles of biological flocks/swarms/schools like firefly and deep-sea fish groups do not send their messages to others all along but just now and then during the whole dynamic process. From the industrial application point of view, the value of this work is two-fold. If the current working condition is in the normal region, then the communication energy or cost can be reduced very sharply at the cost of a tiny decrease of synchronization performance, while in the abnormal region, the minimum communication energy can be estimated to gain the maximum benefit which is larger than the counterpart of complete communication. This work is a first attempt aiming at achieving satisfactory ordered state of a self-propelled particle group via low communication cost, and we believe that it will enlighten the readers on this interesting subject.

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